

## A Abstract

The aim of this proposal is to complete research and follow up with collaborations two of my research projects. Both projects have attracted some international attention, and various co-authors from abroad wish to continue or initiate efforts to obtain more results.

In a nutshell the first of the two projects centers on what is known as 'impact dynamics'. We study the way things 'bounce' off other things. The problem studied by us in great detail is a particle bouncing down a staircase. This kind of mechanics has not really been studied by classical physicists (Newton, etc.), because the process of bouncing, especially where energy is lost, is ill-understood. (Phenomena such as the ones we study are important in for example, industrial processing of granular foodstuff.)

The second project involves the study of the dynamics of a linear network of interacting particles. Also in this case my strategy has been an in-depth study of one case: namely automated traffic on a highway. We imagine each car with sensors that register the relative position and velocity of the car in front of it and the car behind it. The question is now what the best strategy is to have a large number of cars moving at a constant speed. The problem is that the lead car may have to change speed depending on traffic situations. The other cars should react in such a way as to avoid collisions and insure the flow of traffic.

This project constitutes an excellent opportunity for a student to become acquainted with mathematical research, and I ask for a three month GRA stipend. I also ask for release of one course in 2009/10, in order to free up time so that the progress made in research can find its way into published work (thus benefitting both projects), as well as to accompany the student.

I apply for the following support:

1. Student Support (GRA) three months.
2. 1 Course release, for 1 term.
3. Some books and support for departmental expenditures.

## B Background

Let's first have a look at the project involving impact dynamics. When you pour sand, initially a mound will form, and subsequently it will become steeper and steeper. But finally, the slope will stop increasing and the particles poured on the top will tend to roll far down the mound. The equations for any realistic model of such a process are much too complicated to glean substantial analytic insight from. We therefore study a simplified model of this kind of dynamics.

In [9, 2], written during my recent Sabbatical in Spain, we gave a very detailed mathematical treatment of the dynamics of a particle falling down an inclined slope with a simple periodic profile (a "staircase"). The difficulty in this kind of analysis is the following. The traditional theorems in the mathematical study of dynamics require motion to be 'smooth'. Here however, the dynamics is 'bouncy' with 'discontinuities' (namely when the particle lands on the extreme end of a step of the staircase) as well as energy-losses in each bounce. Thus most of that body of knowledge does not apply. Nonetheless the main result of especially [9]